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A VARIATIONAL PRINCIPLE ASSOCIATED WITH A LOCALIZED NUMERICAL SOLUTION OF UNSTEADY FREE-SURFACE FLOWS

DAVID W. TAYLOR NAVAL SHIP RESEARCH AND DEVELOPMENT CENTER

Bethesda, Maryland 20084

A VARIATIONAL PRINCIPLE ASSOCIATED WITH A LOCALIZED NUMERICAL SOLUTION OF UNSTEADY FREE-SURFACE FLOWS

by

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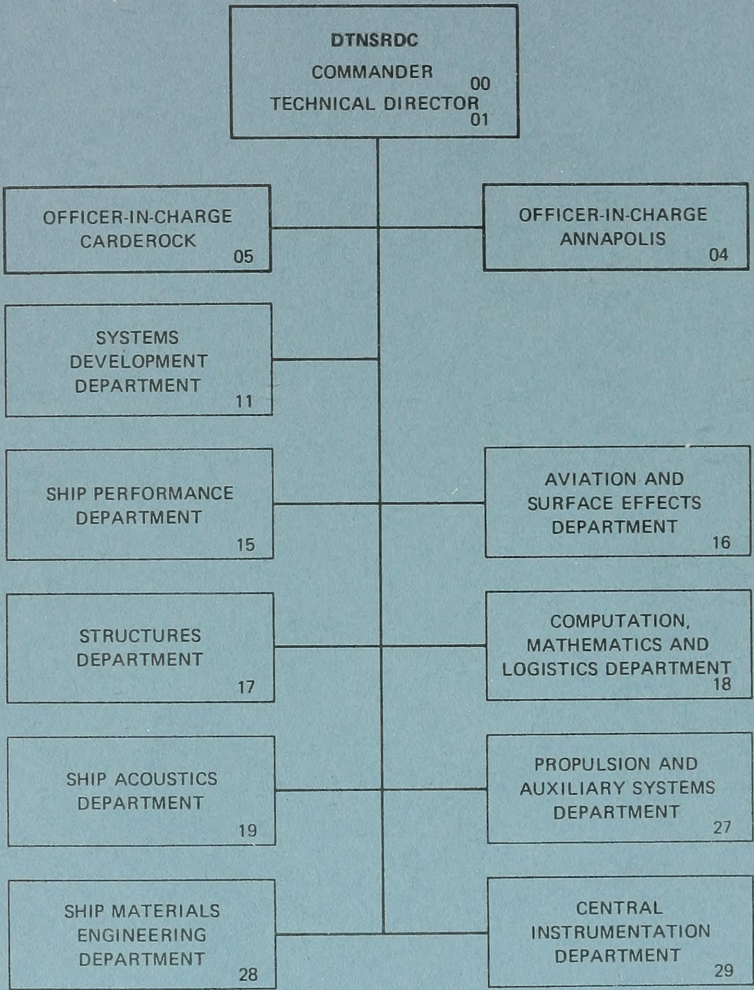

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In this report, a variational principle for unsteady body wave problems is treated both with and without a convolution integral and with both linear and nonlinear free-surface conditions. Functionals are obtained for the numerical computation of unsteady flow fields near a body that moves on or beneath the free surface. This formulation can be applied to ship hydrodynamic performance problems of water entry and body slamming, as well as to arbitrary body motion.

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NOTATION

D	Fluid domain
f_i	Function defined in Equation (23)
g	Acceleration of gravity
g_i	Function defined in Equation (24)
h	Body or free surface defined in Equation (1)
J	Functional
m	Source distribution
p	Pressure distribution
S_F	Free surface
S_J	Interface of near and far fields
S_s	Body surface
t	Time
x,y,z	Rectangular Cartesian coordinates
δ	Variation
ρ	Water density
τ	Time
ϕ	Potential
ψ	Function defined in Equation (21)
ω	Frequency defined in Equation (21)

Subscript

i	The ith order
n	Normal derivative toward fluid
o	Projection on the $z = 0$ surface
t	The time derivative

z	The z derivative
1	Near field, the first order
2	Far field, the second order

ABSTRACT

In this report, a variational principle for unsteady body wave problems is treated both with and without a convolution integral, and with both linear and nonlinear free surface conditions. Functionals are obtained for the numerical computation of unsteady flow fields near a body that moves on or beneath the free surface. This formulation can be applied to ship hydrodynamic performance problems of water entry and body slamming, as well as to arbitrary body motion.

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INTRODUCTION

In the early 1970's the David W. Taylor Naval Ship Research and Development Center (DTNSRDC) recognized the demand for advanced numerical methods to predict the hydrodynamic performance characteristics of naval ships, particularly when classical methods proved inadequate.

Thus, in 1974 the Numerical Naval Ship Hydrodynamics Program was begun at DTNSRDC. Under this program the author previously investigated the steady ship-wave problem using a variational principle associated with a localized finite-element technique.[1*] This method is useful particularly to analyze the flow field near the ship in detail; in the far field, the Michell approximation can be used. This report extends the problem to the unsteady case.

For both the steady and unsteady problems, the simple calculation is for a linear free surface condition with exact body boundary conditions. An iterative method is needed for a nonlinear free surface condition. However, in the unsteady problem, the variational principle requires an integration with respect to time using the initial conditions; in this instance, a convolution integral is useful. The variational principle for the unsteady body wave problem with exact body boundary conditions is treated both with and without convolution, and with both linear and

*A complete listing of references is given on page 13.

nonlinear free surface conditions. For the linear free surface problem, a functional for the variational principle is obtained with a convolution rather than a general integral. The convolution cannot be applied to a nonlinear free surface problem; the condition is required for large values of time. A nonlinear solution derived using an iteration scheme having the linear convolution form is also discussed. The time integration can be eliminated if the motion is sinusoidal.

This formulation can be applied to problems of water entry and body slamming, as well as to arbitrary body motion.

NONLINEAR PROBLEM

Since problems dealt with here can be generalized easily to three-dimensions, for simplicity we first consider a two-dimensional problem in the rectangular Cartesian (x,z) coordinate* plane. When a body whose surface is represented by

$$S_s \; [z=h(x,t)] \tag{1}$$

enters the water surface S_F ($z = 0$, $t \leq 0$; $z = h(x,t)$, $t > 0$) at time $t = 0$, or when a semisubmerged or fully submerged body starts to move at time $t = 0$ and either exits the water or stops moving at $t = t_1$, then the boundary conditions[2] for a velocity potential ϕ are as follows:

$$\phi = \phi_t = 0 \text{ for } t \leq 0 \text{ everywhere}$$

$$\frac{1}{2} (\nabla\phi)^2 - \phi_t + gh = 0 \qquad \text{on } S_F \tag{2}$$

$$\left.\begin{array}{c} h_t - \nabla\phi\nabla\,(h-z) = 0 \\ \text{or} \\ h_t - \phi_n \sqrt{h_x^2+1} = 0 \end{array}\right\} \text{ on } S_F \text{ and } S_s(t) \tag{3}$$

Here, $S_s(t)$ is the submerged body surface varying with time t, and n is the normal direction into the fluid. We consider potentials ϕ_1 in the domain D_1 and ϕ_2 in the

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*Definition of notations are given on page iv.

domain D_2, where D_1 is the near field including S_s, and D_2 is outside of D_1. Then at the interface S_J of D_1 and D_2, we need to have

$$\begin{aligned} \phi_1 &= \phi_2 \\ \phi_{1n} &= -\phi_{2n} \end{aligned} \tag{4}$$

The outer potential ϕ_2 in D_2 is assumed to satisfy the linear free surface condition

$$\phi_{2tt} + g\phi_{2z} = 0 \tag{5}$$

For such ϕ_2 we know the time-dependent Green function.[3,4]

Now we will construct a Lagrangian for the previously described problem, considering the Lagrangian that Luke[5] used

$$J = \int_0^\tau \iint_{D_1} \left(\frac{1}{2} \nabla\phi_1 \nabla\phi_1 - \phi_{1t} \right) dzdxdt + \int_0^\tau \int_{S_{1F}} g \frac{h^2}{2} dxdt - \int_0^\tau \int_{S_J} \left(\phi_1 - \frac{1}{2} \phi_2 \right) \phi_{2n} \, dzdt \tag{6}$$

where ϕ_1, ϕ_2 and h vary with time, and τ is a sufficiently large time after the body has either exited from or come to rest in the water so that we can safely assume that the variation $\delta\phi_1$, $\delta\phi_2$, and $\delta\phi_{2t}$ vanishes at $t = \tau$. It will be shown later that the use of a convolution integral necessitates only the initial condition without the condition at $t = \tau$.

Since

$$\frac{\partial}{\partial t} \int^{h(x,t)} \phi dz = \int^{h(x,t)} \phi_t ds + \phi(z{=}h)\, h_t \tag{7}$$

we have

$$\int_0^\tau \int dx \int^{h(x,t)} \phi_t \, dzdt = \int_0^\tau \int \left[\frac{\partial}{\partial t} \int^{h(x,t)} \phi dz - \phi(z=h)\, h_t \right] dxdt$$

$$= \int \left[\int^{h(x,t)} \phi dz \Bigg|_0^\tau - \int_0^\tau dt\, \phi(z=h)\, h_t \right] dx$$

or

$$\int_0^\tau \iint_{D_1(t)} \phi_t \, dzdxdt = -\int_0^\tau \int_{S_s \cup S_F} h_t \, \phi dxdt + \iint_{D_1(t=\tau)} \phi dxdz \tag{8}$$

Now we take a variation of J in Equation (6), and use the Green theorem, and the condition at large $t = \tau$ obtaining[1]

$$\delta J = \int_0^\tau \int_{S_{1F}} \left(\frac{1}{2} \nabla\phi_1 \, \nabla\phi_1 - \phi_{1t} + gh \right) \delta h \, dxdt$$

$$- \int_0^\tau \iint_{D_1} \nabla^2\phi_1 \, \delta\phi_1 \, dzdxdt - \int_0^\tau \int_{S_{1F} \cup S_s} \left(\phi_{1n} \sqrt{h_x^2+1} - h_t \right) \delta\phi_1 \, dxdt$$

$$- \int_0^\tau \int_{S_J} (\phi_{1n} + \phi_{2n}) \, \delta\phi_1 \, dzdt - \int_0^\tau \int_{S_J} (\phi_1 - \phi_2) \, \delta\phi_{2n} \, dzdt$$

$$+ \frac{1}{2} \int_0^\tau \int_{S_J} (\delta\phi_2 \, \phi_{2n} - \phi_2 \, \delta\phi_{2n}) \, dzdt = 0 \tag{9}$$

However from the identity,

$$\iint_{D_2} \{\nabla^2\phi_2\delta\phi_2-\phi_2\nabla^2(\delta\phi_2)\}\ dxdz = \int_{S_J \cup S_{2F}} \{\phi_{2n}\delta\phi_2-\phi_2(\delta\phi_{2n})\}\ ds = 0$$

and from the linear free surface condition on S_{2F}, Equation (5), and the condition at $t = \tau$, we have

$$\begin{aligned}\int_0^\tau\int_{S_J} (\delta\phi_2\phi_{2n}-\phi_2\delta\phi_{2n})\ dzdt &= -\int_0^\tau\int_{S_{2F}} (\delta\phi_2\phi_{2n}-\phi_2\delta\phi_{2n})\ dxdt \\ &= \frac{1}{g}\int_{S_{2F}}\int_0^\tau (\delta\phi_2\phi_{2tt}-\phi_2\delta\phi_{2tt})\ dtdx = \frac{1}{g}\int_{S_{2F}} (\delta\phi_2\phi_{2t}-\phi_2\delta\phi_{2t})_{t=\tau}\ dx = 0\end{aligned} \tag{10}$$

so that the last integral of Equation (9) vanishes. Since $\delta\phi_1$, $\delta\phi_2$, $\delta\phi_{2n}$, and δh are arbitrary, we obtain from Equations (9) and (10) the corresponding time-dependent, free surface boundary value problem represented by Equations (2)-(4) together with the Laplace equation for ϕ. Therefore, solving the variational problem with the functional J of Equation (6) is equivalent to solving the Laplace equation with the boundary conditions as set forth in Equations (2)-(4), provided we assume that

$$\delta\phi_1 = \delta\phi_2 = \delta\phi_{2t} = 0 \qquad \text{at } t = \tau$$

LINEAR PROBLEM

If we assume a linear free surface condition in both S_{1F} and S_{2F} and keep the exact boundary condition [Equation (3)] on the body surface,

$$\phi_n = h_t / \sqrt{h_x^2+1} \tag{11}$$

then we may use

$$J = \int_0^\tau \iint_{D_1} \frac{1}{2} \nabla\phi_1 \nabla\phi_1 \, dzdxdt - \frac{1}{2g} \int_0^\tau \int_{S_{Fo}} \phi_1 \phi_{1tt} \, dxdt$$

$$+ \int_0^\tau \int_{S_s} h_t \phi_1 \, dxdt - \int_0^\tau \int_{S_J} \left(\phi_1 - \frac{1}{2} \phi_2 \right) \phi_{2n} dsdt \tag{12}$$

where S_{Fo} is the projection of S_F on $z = 0$. When we take the variation of J, we have

$$\delta J = - \int_0^\tau \iint_{D_1} \nabla^2 \phi_1 \delta\phi_1 \, dzdxdt - \int_0^\tau \int_{S_{1Fo}} \delta\phi_1 \left(\phi_{1n} + \frac{1}{g} \phi_{1tt} \right) dxdt$$

$$- \int_0^\tau \int_{S_s} \left(\delta\phi_1 \; \phi_{1n} \; \sqrt{h_x^2+1} \; -h_t \right) dxdt$$

$$- \int_0^\tau \int_{S_J} (\phi_1 - \phi_2) \; \delta\phi_{2n} \, dzdt - \int_0^\tau \int_{S_J} (\phi_{1n} + \phi_{2n}) \delta\phi_1 \, dzdt \tag{12a}$$

Here, in addition to Equation (10), the following equation holds

$$\delta \int_0^\tau \int_{S_{Fo}} \phi_1 \phi_{1tt} \, dxdt = \int_0^\tau \int_{S_{Fo}} \delta\phi_1 \phi_{1tt} \, dxdt + \int_0^\tau \int_{S_{Fo}} \phi_1 \delta\phi_{1tt} \, dxdt$$

$$= 2 \int_0^\tau \int_{S_{Fo}} \delta\phi_1 \phi_{1tt} \, dxdt + \int_{S_{Fo}} (\phi_1 \delta\phi_{1t} - \phi_{1t} \delta\phi_t)_{t=\tau} \, dx = 2 \int_0^\tau \int_{S_{Fo}} \delta\phi_1 \phi_{1tt} \, dxdt \tag{13}$$

Thus, as in the previous section, we can easily derive the corresponding linear free surface boundary value problem using Equation (5) with the exact body boundary condition in Equation (11).

USE OF CONVOLUTION

Equations (6) and (12) are Lagrangians in a time-dependent, two-dimensional space with nonlinear and linear free surface conditions, respectively. They could be localized in space but not in time. Namely, we had to specify the conditions on S_F at $t = 0$ and $t = \tau$ with a sufficiently large τ. On the other hand, we did not require the initial condition $\phi_t = 0$ at $t = 0$. In addition, such time τ when $\phi_1 = 0$ on S_F may be too large[3] for practical use. For linear free surface boundary conditions, we can treat our variational problem in the same way as those who have treated variation principles for linear initial value problems[6,7] using convolutions defined by

$$\phi_1 * \phi_2 \equiv \int_0^\tau \phi_1(x,z,t)\ \phi_2(x,z;\tau-t)\ dt$$

$$\nabla\phi_1 * \nabla\phi_2 \equiv \frac{\partial\phi_1}{\partial x} * \frac{\partial\phi_2}{\partial x} + \frac{\partial\phi_1}{\partial z} * \frac{\partial\phi_2}{\partial z} \tag{14}$$

We change Equation (12) to

$$J = \iint_{D_1} \frac{1}{2}\nabla\phi_1 * \nabla\phi_1\ dzdx - \frac{1}{2g}\int_{S_{Fo}} \phi_1 * \phi_{1tt}\ dx$$

$$+ \int_{S_s} h_t * \phi_1\ dx - \int_{S_J} \left(\phi_1 - \frac{1}{2}\phi_2\right) * \phi_{2n}\ dz \tag{15}$$

If we use the identity relation

$$\phi_1 * \phi_2 = \phi_2 * \phi_1$$

and the initial condition $\phi_1 = \phi_{1t} = 0$ at $t = 0$, instead of the conditions $\phi_1 = 0$ on S_F at $t = 0$ and $t = \tau$ with large τ used in the previous section, we obtain, for any time, τ

$$\delta \int_{S_{Fo}} \phi_1 * \phi_{1tt}\, dx = \int_0^\tau \int_{S_{Fo}} \delta\phi_1(x,t)\, \phi_{1tt}(x,\tau-t)dxdt + \int_0^\tau \int_{S_{Fo}} \phi_1(x,t)\, \delta\phi_{1tt}(x,\tau-t)dxdt$$

$$= 2\int_0^\tau \int_{S_{Fo}} \delta\phi_1(x,t)\, \phi_{1tt}(x,\tau-t)dxdt$$

$$+ \int_{S_{Fo}} \{(\phi_1(x,t)\, \delta\phi_{1t}(x,t-\tau) - \phi_{1t}(x,t)\, \delta\phi_1(x,t-\tau)\}_{t=0}^{\tau}\, dx$$

$$= 2\int_0^\tau \int_{S_{Fo}} \delta\phi_1(x,t)\, \phi_{1tt}(x,\tau-t)dxdt$$

$$= 2\int_{S_{Fo}} \delta\phi_1 * \phi_{1tt}\, dx \tag{16}$$

where τ need not be large, and x represents a point on the free surface $z = 0$. Thus, we obtain as in Equation (12a)

$$\delta J = -\iint_{D_1} \nabla^2\phi_1 * \delta\phi_1\, dzdx - \int_{S_{1Fo}} \delta\phi_1 * \left(\phi_{1n} + \frac{1}{g}\phi_{1tt}\right) dx$$

$$- \int_{S_s} \delta\phi_1 * \left(\phi_{1n}\sqrt{h_x^2+1} - h_t\right) dx$$

$$- \int_{S_J} (\phi_1 - \phi_2) * \delta\phi_{2n} dz - \int_{S_J} (\phi_{1n} + \phi_{2n}) * \delta\phi_1 dz \tag{17}$$

To obtain Equation (17), we used a convolution expression of Equation (16), where we can use the initial condition $\phi_1 = \phi_{1t} = 0$.

Since $\delta\phi_1$, $\delta\phi_2$, and $\delta\phi_{2n}$ are arbitrary, we obtain the corresponding equations for a time-dependent linear free surface boundary value problem that has a unique solution.[3]

If we lift out S_J so that D_1 occupies the entire fluid domain, then the last integral of Equation (15) disappears. The resulting equation appears much simpler than that obtained by Murray[7] due to a simple difference in the treatment of the free surface condition. Equation (15) does not give the wave height as a natural boundary condition, whereas Murray's corresponding equation does. However, from $h = \phi_t/g$ the wave height can be obtained.

If we consider eigen solutions that satisfy only the Laplace equation; the linear free surface condition, Equation (5); and the radiation condition in $D_1 \cup D_2$, then ϕ, which satisfies the body boundary condition in Equation (11), can be derived from Equations (15) and (17) by using

$$J = \int_{S_s} \left(\frac{1}{2}\, \phi_n \sqrt{h_x^2+1} - h_t \right) * \phi dx \tag{18}$$

When we know a functional whose minimum value is attained by the solution, we can find the solution numerically by such methods as the finite-element technique[8] or singularity method.[9]
For example

$$\phi = \sum_{i=1}^{N} m_i\, \phi_i \tag{19}$$

where ϕ_i is the Green function, which is available for this problem for a source distribution on the body surface. The source distribution m_i will be obtained from soltuion of the simultaneous equations,

$$\frac{\partial J}{\partial m_i} = 0 \quad , \; i=1,2,\ldots N \tag{20}$$

SINUSOIDAL MOTION

If we consider a sinusoidal ship oscillation such as $h_t = f\, e^{i\omega t}$ for S_s of Equation (1), we substitute

$$\phi_1 = \psi_1(x,z)\, e^{i\omega t} \tag{21}$$

into Equation (12), integrate with respect to t, and obtain

$$e^{-i\omega t} J = J_1 = \iint_{D_1} \frac{1}{2} \nabla\psi_1 \nabla\psi_1 \, dzdx - \frac{\omega^2}{2g} \int_{S_F} \psi_1^2 \, dx$$

$$+ \int_{S_s} f\psi_1 \, dx - \int_{S_J} \left(\psi_1 - \frac{1}{2}\psi_2\right) \psi_{2n} \, dz \tag{22}$$

This is exactly the same Lagrangian that Bai and Yeung[8] used. Working from Equation (18), we can apply eigen solutions to the whole field by using

$$J = \int_{S_s} \left(\frac{1}{2}\psi_n - F\right) \psi ds$$

where

$$h_t / \sqrt{h_x^2+1} = F\, e^{i\omega t}$$

A similar functional was used by Sao et al.[9] to solve the problem of a heaving oscillation of a dock.

ITERATIVE SCHEME

For problems with the linear free surface condition, we can completely localize the numerical scheme in D_1 with $0 \leq t < \tau$ for any τ with the help of the convolution form. However, for nonlinear problems, the finite-element technique has to rely on an iteration. We may thus use an iterative free surface condition[2] on $z = 0$

$$\phi_{itt} + g\,\phi_{iz} + f_i = 0 \tag{23}$$

$$gh_i - \phi_{it} + g_i = 0 \tag{24}$$

where, for the first order perturbation solution, $f_1 = 0$, $g_1 = 0$ and, for the nth order, f_n and g_n are known functions of ϕ_i of the (n-1)th or the lower order solutions.[2]

Then the Lagrangian for each f_i is

$$J_i = \iint_{D_1} \frac{1}{2} \nabla\phi_{1i} * \nabla\phi_{1i}\, dzdx - \frac{1}{2g} \int_{(z=0)\cap D_1} \phi_{1i} * \phi_{1itt}\, dx$$

$$- \int_{(z=0)\cap D_1} \phi_{1i} * f_i\, dx + \int h_t * \phi_{1i}\, dx$$

$$- \int_{S_J} \left(\phi_{1i} - \frac{1}{2}\phi_{2i}\right) * \phi_{2in}\, dz$$

$$\delta J_i = 0 \tag{25}$$

where the solutions for $1 = 1, 2, \ldots n - 1$ should be used to determine the solution when $i = n$. Equation (24) gives the wave height h for each i.

If we specify a time-dependent, free surface pressure distribution p on the projection S_{so} of S_s to $z = 0$ instead of h_t in S_s, we may use

$$f_1 = g_1 = p/\rho \qquad \text{on } S_{so}$$

in Equations (23) and (24) for the first order, where ρ is the water density.

Although we have discussed the two-dimensional time-dependent problem, Equations (19) through (24) can be extended to the three-dimensional time-dependent problem. That is

$$J = \iiint_{D_1} \frac{1}{2} \nabla\phi_1 * \nabla\phi_1 \, dxdydz - \frac{1}{2g} \iint_{(z=0)\cap D_1} \phi_1 * \phi_{1tt} \, dxdy$$

$$- \iint_{(z=0)\cap D_1} \phi_1 * f_i \, dxdy + \iint h_t * \phi_1 \, dxdy$$

$$- \iint_{S_J} \left(\phi_1 - \frac{1}{2}\phi_2\right) * \phi_{2n} \, ds$$

and so on.

CONCLUDING REMARKS

We have formed functionals with both linear and nonlinear free surface boundary conditions. For the former but not the latter case, we could apply a convolution integral. However, the body boundary condition is satisfied exactly in both cases. In many cases, the flow field near an arbitrary body is of interest, and eigen solutions with linear free surface conditions are known. Even in any large unsteady motion such as ship slamming, the free surface condition for a short period in the beginning may be linear, then the convolution may be applied in the early time period. Especially in the slamming problem, the peak pressure is known to be reached early in the beginning and estimation of the early pressure distribution on the slamming body is required. With this functional, we can find the solution for an arbitrary body numerically by such methods as the finite element technique or singularity method. Thus, a wide application of such functionals can be expected.

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